A small tribute to Gary Goldstein on the occasion of his 70th Birthday

On the validity of the transverse angular momentum sum rule

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October 2010

The power of Gary's early work

IF you have a theory

Theory $\rightarrow$ Amplitudes $\rightarrow$ Observables

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if trying to LEARN about the dynamics

Observables $\rightarrow$ Amplitudes $\rightarrow$ Dynamics

Reactions with spin

MANY different Observables; MANY different Amplitudes

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Great work with Mike Moravcsik on this subject

The transverse angular momentum sum rule

My aim

to counter the myth that one cannot have a sum rule for a transversely polarized nucleon

and

to argue that the Bakker, Leader Trueman Transverse Angular Momentum Sum Rule is correct

Two steps in the derivation of a sum rule

1) Derive an expression for expectation value of angular momentum operators in nucleon state specified by 4-momentum P^μ and covariant spin vector S^μ

$$\langle P, S \,|\, J_i \,|\, P, S \,\rangle \qquad i = 1, 2, 3$$

i.e. show dependence of matrix element on variables P^μ and S^μ

2) As stressed by Jaffe and Manohar: the parton model is a Fock space model. Therefore substitute for the nucleon state its expansion in terms of quark and gluon Fock states and identify the expressions which result.

BLT [PR D70 (2004) 114001] demonstrated:standard expression in literature for for $\langle P, S \,|\, J_i \,|\, P, S \,\rangle$ is correct for nucleons polarized longitudinally, but **incorrect** for the transversely polarized case.

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This is origin of incorrect claim that there cannot exist an angular momentum sum rule for a transversely polarized nucleon.

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Correct result for the forward matrix elements of the angular momentum is

$$\langle P', \boldsymbol{s}|J_i|P, \boldsymbol{s}\rangle = 2P^0(2\pi)^3 \left[\frac{1}{2}\, s_i + i(\boldsymbol{P} \times \boldsymbol{\nabla_P})_i\right] \delta^3(\boldsymbol{P'} - \boldsymbol{P}).$$

and there is then no problem with the transverse case —it is quite analogous to the longitudinal case.

Jaffe-Manohar result has

$$\frac{1}{4Mp^0}\left\{(3p_0^2 - M^2)s_i - \frac{3p_0 + M}{p_0 + M}(\boldsymbol{p}\cdot\boldsymbol{s})p_i\right\} \quad \text{instead of} \quad \frac{1}{2}\, s_i$$

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For longitudinal case exactly same as BLT, but for transverse case, for $p_0 \gg m$

$$\text{J-M} \to \frac{3p_0}{4m} \to \infty$$

so no transverse sum rule.

This is NOT controversial: J-M agree with us!

Thus BLT were able to derive a sum rule relating the transverse spin of the nucleon to the **transverse polarized quark densities $\Delta_T q(x)$ and the transverse orbital angular momentum carried by quarks and gluons,** namely

$$\frac{1}{2} = \frac{1}{2} \sum_{\text{flavours}\, f} \left\{ \int dx\, [\, \Delta_T q^f(x) + \Delta_T \bar{q}^f \,] + \sum_{a=\, q,\, \bar{q},\, G} \langle L_{\boldsymbol{s}_T} \rangle^a \right\}$$

where $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

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where $L_{\boldsymbol{s}_T}$ is the component of $\boldsymbol{L}$ along $\boldsymbol{s}_T$.

NB **sum of quark and antiquark transversity densities, in contrast to the case of the tensor charge of the nucleon, where the difference appears.**

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We shall show that the identification of terms on the RHS is correct.

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For simplicity give the proof only for unpolarized quark density $q(x)$, but the argument holds also for the polarized densities $\Delta q(x)$ and $\Delta_T q(x)$.

The sophisticated expression for the quark correlator $\Phi_{\alpha\beta}(k; P, S)$, for a given flavour, integrated over k with the constraint $x = k^+/P^+$ yields

$$\Phi\alpha\beta(x) = P^+ \int \frac{d\xi^-}{2\pi} \, e^{ixP^+\xi^-} \, \langle P,\, S \,|\, \bar{\psi}_\beta(0) \, \psi_\alpha(0, \xi^-, \mathbf{0}_\perp) \,|\, P,\, S \rangle$$

where

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At leading twist $\Phi\alpha\beta(x)$ expressed in terms of three LT quark distribution functions,

$$\Phi(x) = \frac{1}{2}\, \not{P}\{q(x) - 2\,\lambda\,\Delta q(x)\gamma_5 + \Delta_T q(x)\gamma_5\, \not{S}_\perp\}$$

where $\lambda = \pm 1/2$ is the nucleon helicity.

However, nucleon, mass m, is moving **fast** along OZ so

$$P^\mu \approx (P, 0, 0, P)$$

so that

$$Tr[\gamma^0 \Phi(x)] \approx Tr[\gamma^3 \Phi(x)]$$

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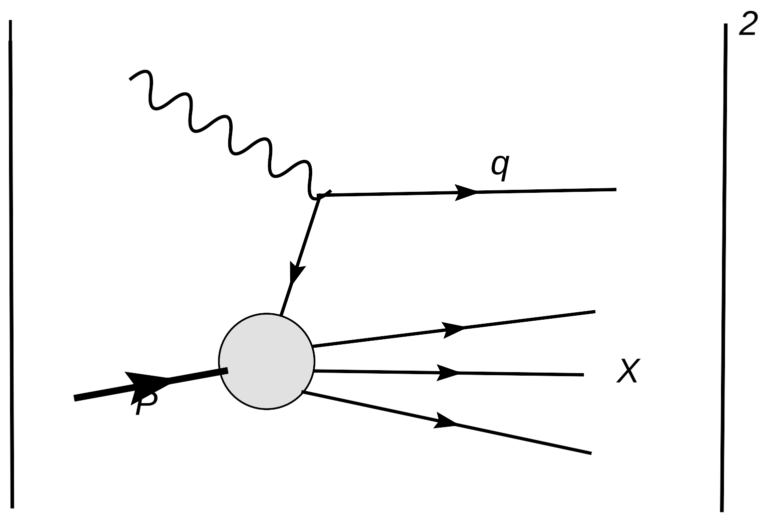
Thus we may take

$$q(x) = \frac{1}{2\, P_0} Tr[\gamma^0 \Phi(x)],$$

Using translation invariance, final expression is

$$q(x) = \frac{1}{\sqrt{2}} \sum_{X,\alpha} |\langle X \,|\, \psi_\alpha(0) \,|\, P,\, S \rangle|^2 \, \delta[P_X^+ - (1-x)P^+]$$

This corresponds to intuitive definition of quark density!



Matrix element involves field at only one time, so evaluate in Interaction Picture i.e use *free-field* expansion

$$\psi_\alpha(0) = \sum_\lambda \int \frac{d^3\boldsymbol{p}}{(2\pi)^3\, 2E_p}\, b(\boldsymbol{p},\lambda) u_\alpha(\boldsymbol{p},\lambda) + \sum_\lambda \int \frac{d^3\bar{\boldsymbol{p}}}{(2\pi)^3\, 2E_{\bar{p}}}\, d^\dagger(\bar{\boldsymbol{p}},\lambda) u_\alpha(\bar{\boldsymbol{p}},\lambda)$$

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where we use $\bar{\boldsymbol{p}}$ to emphasize that $d^\dagger$ creates an antiquark. Note that

$$E_p = \sqrt{\boldsymbol{p}^2 + m_q^2} \qquad \text{and} \qquad E_{\bar{p}} = \sqrt{\bar{\boldsymbol{p}}^2 + m_q^2}$$

The nucleon, $\boldsymbol{P} = (0, 0, P)$, expanded as superposition of n-parton Fock states

$$
\begin{aligned}
|\boldsymbol{P}, m\rangle &= [(2\pi)^3 2P_0]^{1/2} \sum_n \sum_\mu \int \frac{d^3\boldsymbol{k}_1}{\sqrt{(2\pi)^3 2k_1^0}} \cdots \frac{d^3\boldsymbol{k}_n}{\sqrt{(2\pi)^3 2k_n^0}} \\
&\quad \times \psi_{\boldsymbol{P},m}(\boldsymbol{k}_1, \mu_1, \ldots \boldsymbol{k}_n, \mu_n) \\
&\quad \times \delta^{(3)}(\boldsymbol{P} - \boldsymbol{k}_1 - \ldots - \boldsymbol{k}_n) |\boldsymbol{k}_1, \mu_1, \ldots \boldsymbol{k}_n, \mu_n\rangle .
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\end{aligned}
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$\psi_{\boldsymbol{P},m}$ is the partonic wave function of nucleon normalized so that

$$
\sum_{\{\sigma\}} \int d^3\boldsymbol{k}_1..d^3\boldsymbol{k}_n |\psi_{\boldsymbol{P},m}(\boldsymbol{k}_1, \mu_1, ...\boldsymbol{k}_n, \mu_n)|^2 \delta^{(3)}(\boldsymbol{p} - \boldsymbol{k}_1 - ... - \boldsymbol{k}_n) = \mathcal{P}_n
$$

with $\mathcal{P}_n$ the probability of the n-parton state.

Consider contribution to the matrix element $\langle\, X\,|\psi_\alpha(0)|\,P\,\rangle$ when the nucleon is represented by Fock state with n constituents

$$|\,\boldsymbol{k}_1,\mu_1;------\boldsymbol{k}_n,\mu_n\rangle \quad \text{with} \quad \sum_j \boldsymbol{k}_{j_T}=0 \quad \text{and} \quad \sum_j k_{j_z}=P$$

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Show that this is not quite a miracle and that there is no problem also in the Instant form.

Focus on the $d^\dagger$ term in $\psi_\alpha(0)$

$$\langle\, X\,|d^\dagger(\bar{\boldsymbol{p}},\lambda)|\,\boldsymbol{k}_1,\mu_1; - - - - - \boldsymbol{k}_n,\mu_n\rangle \;\; = \;\; \langle\, X\,|\,\bar{\boldsymbol{p}},\lambda :\; \boldsymbol{k}_1,\mu_1; - - - - - \boldsymbol{k}_n,\mu_n\rangle$$

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Aside from combinatoric factors this implies

$$| X \rangle = | \bar{\boldsymbol{p}}, \lambda : \boldsymbol{k}_1, \mu_1; - - - - - \boldsymbol{k}_n, \mu_n \rangle$$

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For this state

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Its energy is

$$\begin{aligned} E_X &= \sum_j \sqrt{\boldsymbol{k}_j^2 + m_q^2} + E_{\bar{p}} \\ &\geq \sum_j k_{j_z} + E_{\bar{p}} \\ &\geq P + E_{\bar{p}} \end{aligned}$$

This implies

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The rest of the derivation is standard giving $q(x)$ as the sum of wave-functions squared for helicity $+1/2$ + helicity $-1/2$.

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The rest of the derivation is standard giving $q(x)$ as the sum of wave-functions squared for helicity $+1/2$ + helicity $-1/2$.

Thus the identification of the wave functions used by BLT with the standard parton densities is correct

First ever use of the transverse sum rule

First moment of u and d transversity from Anselmino et al arXiv:0812.4366 ....assumes sea quark transversity zero

$$J_{Tr} = \frac{1}{2} = 0.16^{+0.07}_{-0.14} + L_{Tr}$$

Compare with

$$J_z = \frac{1}{2} = 0.42 \pm 0.19 + L_z \quad \text{for } \Delta G > 0$$

or

$$J_z = -0.21 \pm 0.46 + L_z \quad \text{for changing sign } \Delta G$$

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The simplification of using instant from wave functions does not affect the partonic interpretation of the terms in the sum rule.

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I wish him many, many more years of fruitful research!

EXTRA SLIDES

A peculiarity concerning the parton orbital angular momentum in a transversely polarized nucleon

In this section we shall draw attention to a peculiarity regarding the quark and gluon *orbital* angular momentum inside a transversely polarized nucleon, which does *not* depend on the precise definition of quark *vs* gluon angular momentum.

The shortest and most direct way to obtain the correct expression for the angular momentum matrix element $\langle P', \boldsymbol{s}|J_i|P, \boldsymbol{s}\rangle$ is actually from consideration of the effect of rotations on a state vector.

But if one uses the traditional approach, via the energy momentum tensor $T^{\mu\nu}$, then to begin with the matrix element of M^{jk}, where

$$J_{(i=x,y,z)} = \epsilon_{ijk} M^{jk} \tag{1}$$

depends on some of the scalar functions appearing in the matrix element of $T^{\mu\nu}$. Using Ji's notation A and B for these, one has

$$\begin{aligned} \langle P, \boldsymbol{s} | M^{ij} | P, \boldsymbol{s} \rangle &= \frac{A}{2M(P_0 + M)} [P^i (\boldsymbol{P} \times \boldsymbol{s})^j - P^j (\boldsymbol{P} \times \boldsymbol{s})^i] \\ &+ \frac{1/2(A+B)}{M} \epsilon^{ij\alpha\beta} S_\alpha P_\beta \end{aligned}$$

Using the fact that T^{00} is the energy density one can show that the energy of the nucleon fixes the value of A so that

$$A(\text{nucleon}) = 1. \tag{2}$$

Applying Eqn. (2) to the case of a longitudinally polarized nucleon with helicity 1/2 implies that $1/2(A+B) = 1/2$. It follows that

$$B(\text{nucleon}) = 0 \tag{3}$$

Suppose now that we split A and B into quark (meaning a sum over flavours of quarks and antiquarks) and gluon pieces:

$$A(\text{nucleon}) = A_q + A_g = 1 \tag{4}$$

$$B(\text{nucleon}) = B_q + B_g = 0 \tag{5}$$

Then the quark and gluon components of M_{ij} will be given by

$$
\begin{aligned}
\langle\ P,\boldsymbol{s}|M^{ij}_{q,g}|P,\boldsymbol{s}\rangle &= \frac{A_{q,g}}{2M(P_0+M)}[P^i(\boldsymbol{P}\times\boldsymbol{s})^j - P^j(\boldsymbol{P}\times\boldsymbol{s})^i] \\
&+ \frac{1/2(A+B)_{q,g}}{M}\epsilon^{ij\alpha\beta}S_\alpha P_\beta \qquad (6)
\end{aligned}
$$

For a transversely polarized nucleon, say along OX, with $\boldsymbol{s}_T=(1,0,0)$ and $\boldsymbol{P}=(0,0,P)$ this becomes

$$
\begin{aligned}
\langle P,\boldsymbol{s}_T|J^q_x|P,\boldsymbol{s}_T\rangle &= \frac{1}{2M}[(M-P_0)A_q + 2P_0\,1/2(A+B)_q] \\
&= \frac{1}{2M}[MA_q + P_0\,B_q] \qquad (7)
\end{aligned}
$$

Thus for a fast moving nucleon, as $P_0 \to \infty$, J_x^q, and similarly J_x^g, becomes infinite. Of course J_x^{nucleon} is finite, since, as mentioned above, $B_q + B_g = 0$.

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Thus irrespective of how the angular momentum is split into quark and gluon components, the separate quark and gluon *orbital* angular momentum in a fast moving transversely polarized nucleon becomes infinite. Their sum, however, is finite.